Interacting Electron-Photon System

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Outline

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2. Single Photon System
3. Single Electron System
4. Two-body, Non-interacting System
5. Two-body, Interacting System
Introduction

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- In Bohmian Mechanics, particles have definite positions that change with time. One wave function, defined on the configuration space of a system of particles, guides the motion of all particles through their respective guiding equations.

First we examined the system of a single photon, then that of a single electron, then that of the two without any interaction, and, at last, the system of the photon and electron interacting.
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- In one space dimension this means:

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- Since photon is a relativistic quantum particle, its wave function must satisfy a relativistic (PDE) equation.
- Kiessling & Tahvildar-Zadeh also discovered this equation in 2018. It is a Dirac-type equation, and in 1-dim. it reads:
  \[-i\hbar \gamma^\mu \frac{\partial \Psi_{ph}}{\partial x^\mu} = 0,\]
  where \(\hbar = \) reduced Planck’s constant, \(x^0 = t, x^1 = s\),
  \(\gamma^0 = \begin{pmatrix}
  0 & 1 \\
  1 & 0
  \end{pmatrix}, \gamma^1 = \begin{pmatrix}
  0 & -1 \\
  1 & 0
  \end{pmatrix}\), and repeated indices are summed over the range \(\mu = 0, 1\).
Initial Wave Function

- The photon wave equation needs to be solved given an initial wave function $\psi_{ph}(0, s) = \psi_{ph}^0(s)$. 

Typical initial data corresponding to a photon localized in both position and momentum space (subject to Uncertainty Principle) are pictured below:
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According to Kiessling and Tahvildar-Zadeh, in one space dimension the quantum probability current of detecting the photon is

\[ j_{\mu}^{\text{ph}}(\text{time}, \text{position}) = \frac{1}{4} \text{trace}(\overline{\Psi}_{\text{ph}} \gamma^\mu \Psi_{\text{ph}} \gamma(X)) \]
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Here \( X = (X^0, X^1) \) is a constant vector field computed from \( \psi_{ph}^0 \), \( \gamma(X) := \gamma_0 X^0 + \gamma_1 X^1 \), and \( \overline{\Psi} := \gamma^0 \Psi^\dagger \gamma^0 \).
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The probability density of detecting the photon at event \((t, s)\) is \( \rho(t, s) = j^0_{\text{ph}}(t, s) \).
Photon Probability Density

- The photon probability density looks like this:
  http://reu.dimacs.rutgers.edu/~aas377/photon_pdf.mp4
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- Varying initial conditions gives us the following: http://reu.dimacs.rutgers.edu/~aas377/multiple_photon_pdf.mp4
The Guiding Equation

- The motion of the photon is guided by its wave function:

\[
\begin{align*}
\frac{dq}{dt} &= v_{ph}(t, q(t)) = \frac{j^1(t, q(t))}{j^0(t, q(t))} \\
q(0) &= q_0
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- \( q_0 \) is the actual initial position of the photon. All we know about it is that it is randomly distributed according to the initial probability density \( \rho(0, s) \).
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Single Electron System

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- In one space dimension this means:

\[ \Psi_{el}(t, s) = \begin{pmatrix} \Psi_-(t, s) \\ \Psi_+(t, s) \end{pmatrix} \]

Like in the case of a single photon, the wave function of a single electron also satisfies a relativistic equation. In particular, it satisfies the massive Dirac equation:

\[ -i\hbar \gamma^\mu \partial_\mu \Psi_{el} + m_{el} \Psi_{el} = 0, \]

where \( m_{el} \) is the mass of electron.
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-\frac{i\hbar}{m_{el}} \gamma^\mu \partial_\mu \Psi_{el} + m_{el} \Psi_{el} = 0,
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where \( m_{el} = \) the mass of electron.
Electron Probability Current and Velocity Field, and Guiding Equation

- The probability current of an electron is known:

\[ j'^{\mu}_{el}(time, position) = \overline{\Psi}_{el} \gamma^{\mu} \Psi_{el}, \]

where \( \overline{\Psi} := \Psi^\dagger \gamma^0 \) is the Dirac adjoint for rank-one bispinors.
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- We define the guiding velocity field of an electron in the same way as that of a photon:

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- We define the guiding velocity field of an electron in the same way as that of a photon:

\[ v_{el}(t, s) := \frac{j^1(t, s)}{j^0(t, s)}. \]

- Similarly to the photon case, the guiding equation for the electron is:

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\begin{cases}
    \frac{dq}{dt} = v_{el}(t, q(t)) = \frac{j^1(t, q(t))}{j^0(t, q(t))} \\
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Electron Probability Density

http://reu.dimacs.rutgers.edu/~aas377/electron_pdf.mp4
Electron Trajectories and Parameters

- The electron wave function $\Psi_{el}$, has a mass term: $\omega = \text{mass}/\hbar$, and a parameter we can change: standard deviation of the initial distribution: $\sigma$. The following graph shows the trajectory of an electron guided by the velocity field with different parameters.
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![Graph showing electron trajectories with different parameters](image)

1. For $\sigma = 1$, $\omega = 1$, trajectories are more dispersed.
2. For $\sigma = 0.5$, $\omega = 1$, trajectories are less dispersed compared to the previous case.
3. For $\sigma = 0.1$, $\omega = 1$, trajectories are the most focused, indicating a higher standard deviation of the initial distribution.

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Two-body, Non-interacting System

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- The wave function, $\psi$, is a function of four variables, namely the time and position of each particle.
- To get a wave function that describes both a photon and an electron in a non-interacting system, we take the Tensor Product ($\otimes$) of the electron and the photon wave functions, giving us a four component object $\psi = (\psi_{++}, \psi_{+-}, \psi_{-+}, \psi_{--})$. 
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- The guiding equations for photon and electron are derived using the Hypersurface Bohm-Dirac (HBD) Theory, which allows us to describe the motion of the photon and electron in a common time.
Wave Equation and Probability Current

- The tensored wave function satisfies a relativistic wave equation obtained by Tensor Product of the photon and electron wave equations

\[
\begin{cases}
- \imath \hbar \gamma^\mu \partial x^\mu_{ph} \psi = 0 \\
- \imath \hbar \gamma^\mu \partial x^\mu_{el} \psi + m_{el} \psi = 0 \\
\psi(0, s_{ph}, 0, s_{el}) = \psi^0(s_{ph}, s_{el})
\end{cases}
\]

Since Tensor Product preserves probability distributions, “multiplying” the probability density movies of the photon and electron gives the joint probability density for the non-interacting system:

http://reu.dimacs.rutgers.edu/~aas377/non_interacting2.mp4

The probability current is the following:

\[
\begin{array}{l}
\begin{align*}
J_{00} &= |\psi^{++}|^2 + |\psi^{+-}|^2 + |\psi^{-+}|^2 + |\psi^{--}|^2 \\
J_{10} &= |\psi^{++}|^2 - |\psi^{+-}|^2 - |\psi^{-+}|^2 + |\psi^{--}|^2 \\
J_{01} &= |\psi^{++}|^2 - |\psi^{+-}|^2 + |\psi^{-+}|^2 - |\psi^{--}|^2 \\
J_{11} &= |\psi^{++}|^2 - |\psi^{+-}|^2 - |\psi^{-+}|^2 - |\psi^{--}|^2
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  j^{00} \\
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  |\psi_{++}|^2 + |\psi_{+-}|^2 + |\psi_{-+}|^2 + |\psi_{--}|^2 \\
  |\psi_{++}|^2 + |\psi_{+-}|^2 - |\psi_{-+}|^2 - |\psi_{--}|^2 \\
  |\psi_{++}|^2 - |\psi_{+-}|^2 + |\psi_{-+}|^2 - |\psi_{--}|^2 \\
  |\psi_{++}|^2 - |\psi_{+-}|^2 - |\psi_{-+}|^2 + |\psi_{--}|^2
\end{pmatrix}
\]
Trajectories of the Two-body, Non-interacting System

- The following graph shows the trajectories of a non-interacting system of one electron and one photon.
To obtain an interacting system from a non-interacting system, it is necessary to add some conditions such that the particles do not simply go through each other.
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We do this by adding a boundary condition: we set the relative velocities of photon and electron to be 0 when the particles are at the same space and time.
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We do this by adding a boundary condition: we set the relative velocities of photon and electron to be 0 when the particles are at the same space and time.

Adding the boundary condition to the wave function gives us a modified probability density function: http://reu.dimacs.rutgers.edu/~aas377/interacting_pdf_2.mp4
Adding the boundary condition to the wave function gives us the trajectories of an interacting electron-photon system.
Varying Parameters in the Interacting System

- Changing the sigma and omega of the electron gives us the following changes in trajectories of the electron:
Varying Parameters in the Interacting System

- Changing the polarization angles of electron and photon gives us:
Varying Parameters in the Interacting System

- Changing the mean momentum of the incoming photon varies the trajectories as follows:

![Graphs showing trajectories for different photon momenta](image-url)
Varying Parameters in the Interacting System

- Changing the mean momentum of the incoming photon varies the trajectories as follows:

- Momentum is related to energy, so if the photon does not have enough momentum, it cannot get the electron to bounce away.
Future Steps

- We need to continue performing numerical experiments in order to clarify the roles of different parameters.
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- In order to verify our framework, we need to find a way to calculate momentum after the photon-electron collision and compare our findings to the results of Compton Scattering in one space dimension.
Thank You

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- For more information, visit us at CoRE 417!