Interacting Electron-Photon System

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Outline

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- In Bohmian Mechanics, particles have definite positions that change with time. One wave function, defined on the configuration space of a system of particles, guides the motion of all particles through their respective guiding equations.
- First we examined the system of a single photon, then that of a single electron, then that of the two without any interaction, and, at last, the system of the photon and electron interacting.

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- Kiessling & Tahvildar-Zadeh also discovered this equation in 2018. It is a Dirac-type equation, and in 1-dim. it reads:

$$-i\hbar\gamma^\mu \frac{\partial \Psi_{ph}}{\partial x^\mu} = 0,$$

where $\hbar =$ reduced Planck's constant, $x^0 = t, x^1 = s$, $\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\gamma^1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, and repeated indices are summed over the range $\mu = 0, 1$.

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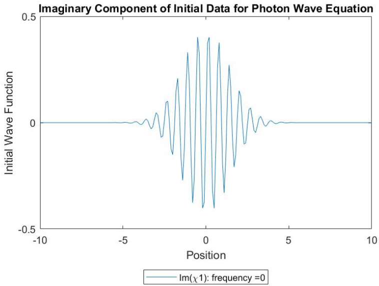
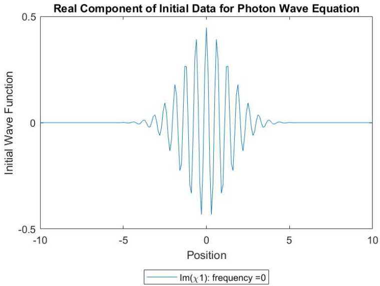
- Typical initial data corresponding to a photon localized in both position and momentum space (subject to Uncertainty Principle) are pictured below:





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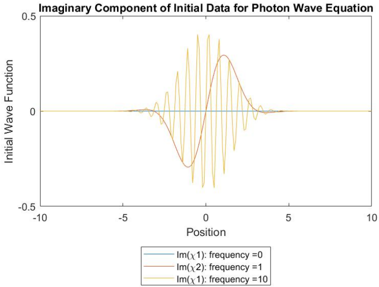

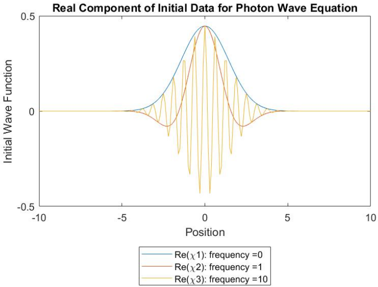

Photon Probability Current and Velocity Field

- According to Kiessling and Tahvildar-Zadeh, in one space dimension the quantum probability current of detecting the photon is

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- Here $X = (X^0, X^1)$ is a constant vector field computed from Ψ_{ph}^0, $\gamma(X) := \gamma_0 X^0 + \gamma_1 X^1$, and $\overline{\Psi} := \gamma^0 \Psi^{\dagger} \gamma^0$.

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- The current is conserved: $\partial_{\mu} j^{\mu}_{ph} = 0$, future directed ($j^0 \geq 0$), and timelike ($j^0 \geq |j^1|$).
- The probability density of detecting the photon at event (t, s) is $\rho(t, s) = j^0_{ph}(t, s)$.

Photon Probability Density

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- Varying initial conditions gives us the following: http://reu.dimacs.rutgers.edu/~aas377/multiple_photon_pdf.mp4

The Guiding Equation

- The motion of the photon is guided by its wave function:

$$\begin{cases} \frac{dq}{dt} = v_{ph}(t, q(t)) = \frac{j^1(t,q(t))}{j^0(t,q(t))} \\ q(0) = q_0 \end{cases}$$

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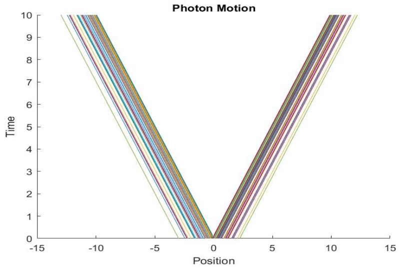
- q_0 is the actual initial position of the photon. All we know about it is that it is randomly distributed according to the initial probability density $\rho(0, s)$.



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- Like in the case of a single photon, the wave function of a single electron also satisfies a relativistic equation. In particular, it satisfies the massive Dirac equation:

$$-i\hbar\gamma^\mu \partial_\mu \Psi_{el} + m_{el}\Psi_{el} = 0,$$

where m_{el} = the mass of electron.

Electron Probability Current and Velocity Field, and Guiding Equation

- The probability current of an electron is known:

$$j^{\mu}_{el}(time, position) = \overline{\Psi_{el}}\gamma^{\mu}\Psi_{el},$$

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- Similarly to the photon case, the guiding equation for the electron is:

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Electron Probability Density

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Electron Trajectories and Parameters

- The electron wave function Ψ_{el}, has a mass term: $\omega =$mass$/\hbar$, and a parameter we can change: standard deviation of the initial distribution: σ.The following graph shows the trajectory of an electron guided by the velocity field with different parameters.

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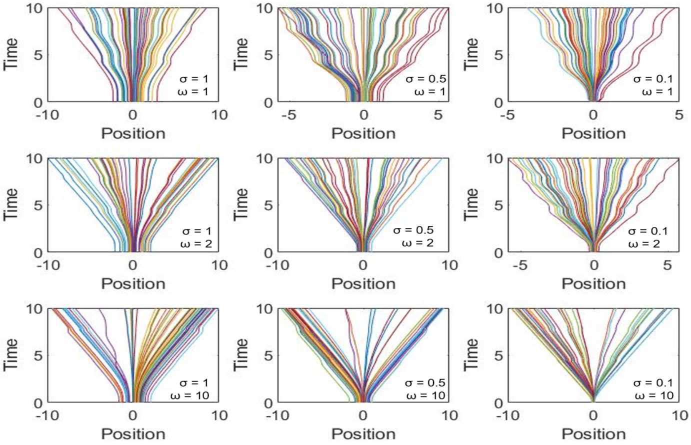

Two-body, Non-interacting System

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- To get a wave function that describes both a photon and an electron in a non-interacting system, we take the Tensor Product ($\otimes$) of the electron and the photon wave functions, giving us a four component object $\psi = (\psi_{++}, \psi_{+-}, \psi_{-+}, \psi_{--})$

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- The guiding equations for photon and electron are derived using the Hypersurface Bohm-Dirac (HBD) Theory, which allows us to describe the motion of the photon and electron in a common time

Wave Equation and Probability Current

- The tensored wave function satisfies a relativistic wave equation obtained by Tensor Product of the photon and electron wave equations

$$\begin{cases} -i\hbar\gamma^{\mu}\partial x^{\mu}_{ph}\psi = 0 \\ -i\hbar\gamma^{\mu}\partial x^{\mu}_{el}\psi + m_{el}\psi = 0 \\ \psi(0, s_{ph}, 0, s_{el}) = \psi^{0}(s_{ph}, s_{el}) \end{cases}$$

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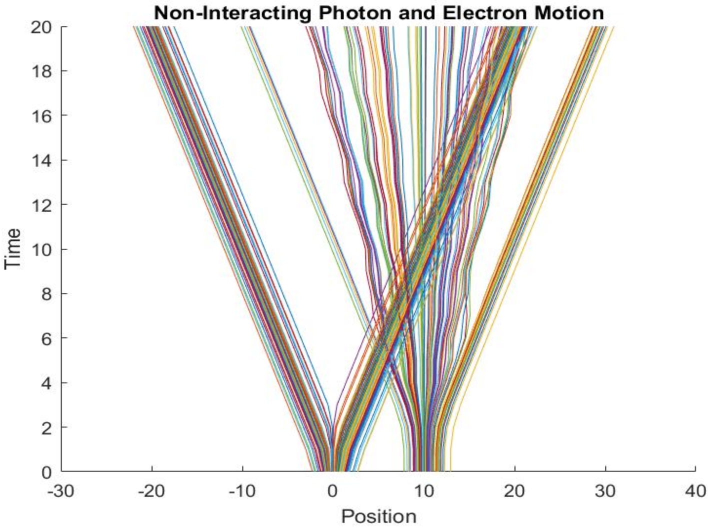
- The probability current is the following:

$$\begin{pmatrix} j^{00} & = & |\psi_{++}|^2 + |\psi_{+-}|^2 + |\psi_{-+}|^2 + |\psi_{--}|^2 \\ j^{10} & = & |\psi_{++}|^2 + |\psi_{+-}|^2 - |\psi_{-+}|^2 - |\psi_{--}|^2 \\ j^{01} & = & |\psi_{++}|^2 - |\psi_{+-}|^2 + |\psi_{-+}|^2 - |\psi_{--}|^2 \\ j^{11} & = & |\psi_{++}|^2 - |\psi_{+-}|^2 - |\psi_{-+}|^2 + |\psi_{--}|^2 \end{pmatrix}$$

Trajectories of the Two-body, Non-interacting System

- The following graph shows the trajectories of a non-interacting system of one electron and one photon.



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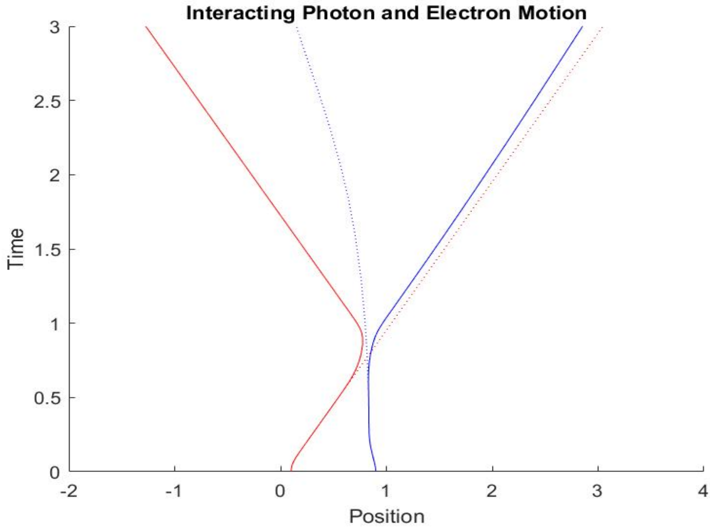
- Adding the boundary condition to the wave function gives us a modified probability density function: `http://reu.dimacs.rutgers.edu/~aas377/interacting_pdf_2.mp4`

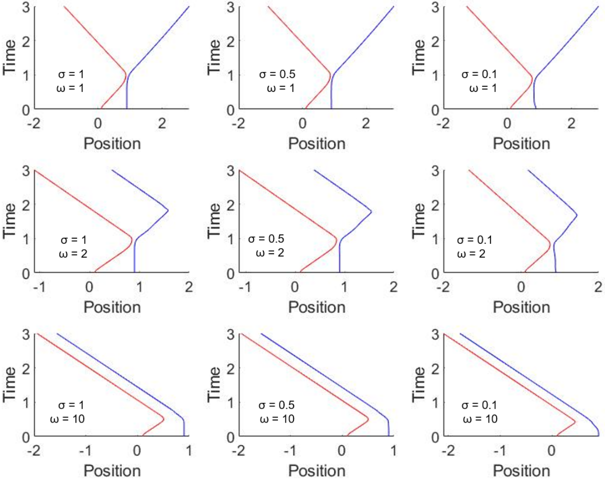
Trajectories of the Two-body, Interacting System

- Adding the boundary condition to the wave function gives us the trajectories of an interacting electron-photon system.



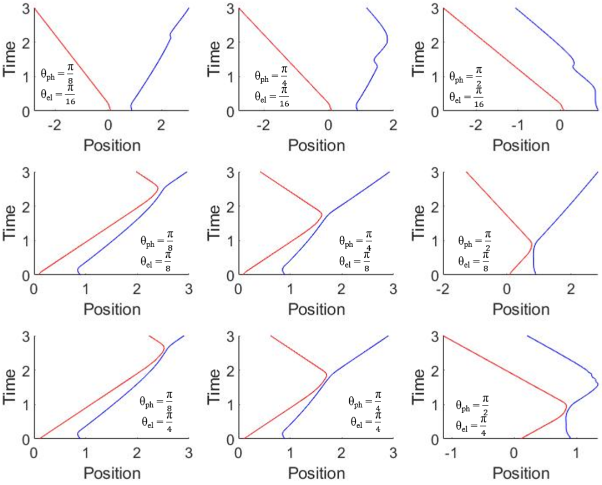
Varying Parameters in the Interacting System

- Changing the sigma and omega of the electron gives us the following changes in trajectories of the electron:



Varying Parameters in the Interacting System

- Changing the polarization angles of electron and photon gives us:



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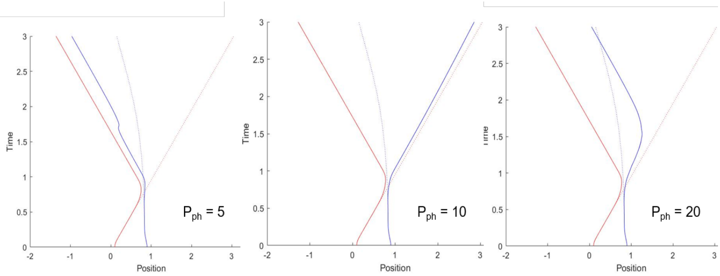
- Changing the mean momentum of the incoming photon varies the trajectories as follows:



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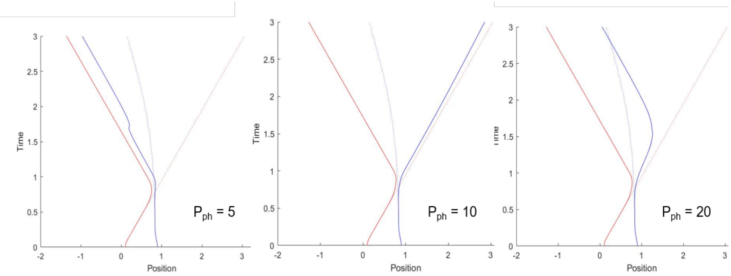

- Momentum is related to energy, so if the photon does not have enough momentum, it cannot get the electron to bounce away.

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- In order to verify our framework, we need to find a way to calculate momentum after the photon-electron collision and compare our findings to the results of Compton Scattering in one space dimension

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- For more information, visit us at CoRE 417!